APPORTS DE LA RECHERCHE

Possibilistic methods for uncertainty treatment

An application to maintenance modelling

Enrico Zio and Nicola Pedroni

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TOPIC

Risk analysis

Les Cahiers de la sécurité industrielle

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http://www.FonCSI.org/

6 allée Émile Monso — BP 34038
31029 Toulouse cedex 4
France

Telephone: +33 534 32 32 00
Twitter: @TheFonCSI
Email: contact@FonCSI.org

Résumé

Titre	Méthodes possibilistes de propagation d'incertitude pour l'analyse de politiques de maintenance
Mots-clefs	incertitude, probabilités, théorie des possibilités, analyse de risque, maintenance
Auteurs	Enrico Zio et Nicola Pedroni
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Les auteurs proposent une méthode pour évaluer la performance d'une politique de maintenance en intégrant l'effet de l'incertitude affectant différents paramètres d'un modèle de dégradation. La méthode proposée est adaptée à la représentation et la propagation d'incertitude de nature épistémique provenant d'avis d'experts. Pour chaque paramètre incertain, l'expert fournit une famille d'intervalles de confiance. Cette information élicitée est représentée à l'aide de **distributions de possibilité** et propagée dans le modèle de dégradation à l'aide de variables aléatoires floues et de la théorie des croyances de Dempster-Shafer.

Les approches classiques de la propagation d'incertitude, basées sur la théorie classique de la probabilité, utilisent des distributions de probabilité pour représenter l'information provenant d'experts. Cependant, les jugements d'expert sont souvent obtenus à l'aide d'expressions linguistiques imprécises, et imposer des distributions de probabilité sur-contraint cette information incertaine de façon arbitraire. La théorie des possibilités permet de représenter cette incertitude épistémique de manière rigoureuse, sans introduire des biais.

Une étude de cas pratique, concernant la maintenance d'une soupape dans une turbo-pompe d'un système de lubrification d'une centrale nucléaire de production d'électricité, permet d'illustrer la méthode. Un modèle de défaillances par rupture provoquées par la fatigue est développé, et une politique de maintenance conditionnelle (basée sur l'état) est appliquée au composant sur un horizon temporel fixe. La performance de la politique de maintenance est évaluée en termes de coûts et d'indisponibilité du composant.

La méthode produite des distributions de plausibilité et de croyance pour les variables de sortie d'intérêt. Des travaux complémentaires sont nécessaires pour aider les décideurs à interpréter ces informations et les intégrer à des processus et outils d'aide à la décision.

About the authors

Enrico Zio est Directeur de la Chaire Systèmes complexes et défis énergétiques de l'École Centrale Paris & Supelec. Il est également chairman du *European Safety and Reliability Association* (ESRA).

Nicola Pedroni est maître de conférences au département Énergie du *Politecnico di Milano*. Sa recherche concerne les méthodes calculatoires avancées pour l'évaluation de la sécurité des systèmes industriels, en présence d'incertitude.

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Abstract

Title Possibilistic methods for uncertainty treatment applied to maintenance policy assessment

Keywords uncertainty, possibility theory, propagation, maintenance policies

Authors Enrico Zio and Nicola Pedroni

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The authors propose a method for assessing the performance of a maintenance policy whilst accounting for uncertainty in various parameters of the degradation model. The method is appropriate for the representation and propagation of epistemic uncertainty which is elicited from an expert, who can provide a family of confidence intervals for each uncertain parameter. Information elicited from the expert is described using **possibility distributions** and propagated through the degradation model using fuzzy random variables and the Dempster-Shafer Theory of Evidence.

In classical approaches to uncertainty propagation based on probability theory, probability distributions are used to represent information obtained from experts. However, expert judgment is often expressed using imprecise linguistic statements, and the imposition of specific probability distributions over-constrains this uncertain information in an arbitrary and unjustified manner. Possibility theory allows the epistemic uncertainty arising from expert opinion to be represented in an arguably more rigorous manner, without introducing additional bias.

A practical case study concerning the maintenance of a check valve of a turbo-pump lubricating system in a nuclear power plant illustrates the method. A rupture failure model caused by fatigue is modeled, and a Condition-Based Maintenance policy is applied to the component over a fixed time horizon. The performance of the maintenance policy is assessed in terms of cost and component unavailability.

The method produces plausibility and belief distributions for the output values of interest. Further work is necessary to help decision-makers interpret this information and integrate it in decision-support procedures.

About the authors

Enrico Zio is Director of the Chair in Complex Systems and the Energetic Challenge of École Centrale Paris & Supelec. He is also chairman of the *European Safety and Reliability Association* (ESRA).

Nicola Pedroni is an associate professor in the Energy Department of the Politecnico di Milano. His research concerns advanced computational methods for the reliability assessment of industrial systems in presence of uncertainties.

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Contents

Introduction

Context: uncertainty analysis in maintenance modeling

Processing of uncertainty is crucial in industrial applications and consequently in decision-making processes [Soares et al. 2010]. In practice, it is often convenient to distinguish uncertainty due to the inherent variability of the phenomena of interest from that due to lack of precise knowledge [Apostolakis 1990]. The former type is referred to as aleatory, irreducible, stochastic or random uncertainty and describes the inherent variation associated with the physical system or the environment, the latter is referred to as epistemic, subjective or reducible uncertainty, and relates to the lack of precise knowledge of quantities or processes of the system or the environment. Although probability theory is well suited to handle stochastic uncertainty due to variability, it has been argued that the classical probabilistic approach may have some limitations in the representation and treatment of epistemic uncertainty in situations of poor knowledge, since it tends to force assumptions which may not be justified by the available information [Baudrit et al. 2008]. For example, ignoring whether a value of a parameter is more or less probable than any other value within a given range does not justify assuming a uniform probability distribution, which is the least informative probability distribution according to both the principle of insufficient reason[1] and the maximum entropy criterion [Dubois 2006].

In this work, we consider alternative approaches to classical probability theory for the representation of epistemic uncertainty, such as Dempster-Shafer Theory of Evidence (DSTE) and possibility theory. These approaches have been considered due to their ability to handle the uncertainty associated with the imprecise knowledge on the values of parameters used by expert information systems and for which reliable data are lacking.

The strength of DSTE and possibility theory lies in their ability to represent the epistemic uncertainty in a less committed manner than that offered by probability theory. Possibility theory has been embraced to tackle a number of interesting issues pertaining to different fields such as graph theory [Borgelt et al. 2000], database querying [Bosc and Prade 1997], diagnostics [Cayrac et al. 1996], data analysis [Wolkenhauer 1998] and classification [Benferhat and Tabia 2009], agricultural sciences [Clarke et al. 1992], probabilistic risk assessment (*e.g.* [Baraldi and Zio 2008a,b]), to cite a few. Analogously, applications of DSTE can be found in diverse domains such as signal and image processing [Bloch 1996], business decision-making [Srivastava and Mock 2002], pattern recognition [Parikh et al. 2001], clustering [Schubert 1997], etc.

In spite of the liveliness of the research in the field, it seems fair to say that the non-probabilistic treatment of uncertainty within soft computing methods has not been properly investigated. After all, given the relative immaturity and small size of research community dedicated to the non-probabilistic approaches, it is hardly fair to expect that these are elaborated from soft methods to the same extent as probability theory [Hall 2006]. In this respect, to the authors' knowledge, possibility theory has never been applied in the context of maintenance modeling, which is the subject of this document.

Maintenance is a key factor for safety, production, asset management and competitiveness. Establishing an optimal maintenance policy requires the availability of logic, mathematical and computational models for:

1. the evaluation of **performance indicators** characterizing a generic maintenance policy. Possible performance indicators are production profit, system mean availability and maintenance costs.
2. the identification of the **optimal maintenance intervention policy** from the point of view of the identified performance indicators, while fulfilling constraints such as those regarding safety and regulatory requirements. In practice, this multi-objective

multi-objective optimization under uncertainty

[1] The principle of insufficient reason is a rule for assigning epistemic probabilities. Suppose that there are $n > 1$ mutually exclusive possibilities (which are collectively exhaustive). The principle of insufficient reason states that if the n possibilities are indistinguishable except for their names, then each possibility should be assigned a probability equal to $1/n$. This principle was first enunciated by mathematicians Bernouilli and Laplace.

optimization problem has to be faced in a situation in which some constraints and/or the objective functions are affected by uncertainty. To effectively tackle this problem, a number of approaches have been already proposed in the literature considering different frameworks for uncertainty representation: probability distributions in [Deb et al. 2009; Eskandari et al. 2007; Hughes 2001], fuzzy sets in [Li and Kwan 2003; Trebi-Ollennu and White 1997], and plausibility and belief functions in [Limbourg 2005].

Objectives of this document

This document contributes to step 1) identified above, by developing a methodology for maintenance performance assessment that properly processes the involved uncertainties. More specifically, we consider a situation in which:

- ▷ A stochastic model of the life of the component of interest, in terms of degradation process, failure behavior and maintenance interventions is known without any uncertainty. This is, for example, the case for the degradation process 'fatigue' which has been successfully modeled by means of gamma processes [van Noortwijk 2009], Weibull distributions [Wormsen and Härkegård 2004], Paris-Erdogan law [Paris and Erdogan 1963], etc.
- ▷ The model of the component's behavior depends on a number of ill-known parameters. With reference to the example of fatigue degradation, the gamma process, Weibull distribution and Paris-Erdogan law depend on parameters whose values are usually not precisely known. Moreover, knowledge of other model parameters such as those describing the maintenance effectiveness (*e.g.*, the improvement of the component degradation), duration and cost may also be imprecise.
- ▷ Information about the ill-known parameters is available only from experts; in particular, it is assumed that there is a single expert, who provides for every uncertain parameter a set of intervals, which contain its true value with different degrees of possibility.

Although methods for evaluating *a priori* the performance of a maintenance policy while taking into account the aleatory uncertainty on the future behavior of the component of interest have been investigated in the literature (see [Wang 2002; Singpurwalla 1995; Valdez-Flores and Feldman 1989; Wang and Pham 1999] for surveys), only few works (*e.g.*, [Nicolai et al. 2009]) tackle the maintenance policy performance assessment problem considering the epistemic uncertainty on the maintenance model parameters. In this work, the information elicited from the expert is described by means of possibility distributions and propagated through the model by resorting to a method that exploits the concept of FRVs[2] [Baudrit et al. 2008; Shapiro 2009] and the DSTE.

The method is illustrated with reference to an exponential, non-repairable, binary component. A practical case study is shown with reference to the degradation model of a check valve of a turbo-pump lubricating system in a nuclear power plant.

Document structure

The document is organized as follows:

- ▷ Chapter 1 starts with a reminder of basic concepts of possibility theory and the Dempster-Shafer theory of evidence.
- ▷ Chapter 2 illustrates the method used to elicit, represent and propagate the uncertainties.
- ▷ Chapter 3 illustrates a case study, which is for reference first investigated assuming that there is no epistemic uncertainty affecting the parameters of the stochastic model. The FRV-based method is then applied to this case study to treat epistemic uncertainty.
- ▷ Finally, a discussion of the pros and cons of the method, emerging from its application to the case study, concludes the document.

[2] FRV: Fuzzy random variable

Readers may be interested by a number of other documents by the same authors in the collection of the *Industrial Safety Cahiers*:

- ▷ *Uncertainty characterization in risk analysis for decision-making practice* (CSI-2012-07), which provides an overview of sources of uncertainty which arise in each step of a probabilistic risk analysis;
- ▷ *Overview of risk-informed decision-making processes* (CSI-2012-10), which illustrates the way in which NASA and the US Nuclear Regulatory Commission implement risk-informed decision-making.
- ▷ *Literature review of methods for representing uncertainty* (CSI-2013-03), which provides an overview of probability theory, interval analysis and possibility theory and their use for risk analysis.
- ▷ *Case studies in uncertainty propagation and importance measure assessment* (CSI-2013-12) applies possibilistic methods to the estimation of importance measures for components in the presence of epistemic uncertainty and to the propagation of uncertainty in a flood risk model.

1

Theory overview

This chapter provides a very brief introduction to the Dempster-Shafer theory of evidence and to possibility theory, the uncertainty modelling methods used in this document. Another Cahier titled *Literature review of methods for representing uncertainty*, published by the authors in the same collection as this document (n° 2013-03), provides more detail on these two methods and compares them with classical probabilistic analysis, imprecise probability (interval analysis) and probability bound analysis in a risk-analysis setting [Zio and Pedroni 2013].

1.1 Dempster-Shafer Theory of Evidence (DSTE)

Belief functions can be used to process information which is at the same time of random and imprecise nature. The related formal DSTE (also called Theory of Belief Functions) involves the specification of a triplet (S, I, m), where S (called 'sample space') is the set that contains everything that could occur in the universe under consideration, I (referred to as 'set of focal elements') is a countable collection of subsets of S, and m (Basic Probability Assignment, BPA) is a function defined on subsets of S such that:

$$m(E) = \begin{cases} > 0 & \text{if } E \in I \\ 0 & \text{if } E \notin I \text{ and } E \subset S \end{cases}$$

and

$$\sum_{E \in I} m(E) = 1$$

More intuitively, the DSTE assigns weights (probability masses) to the focal sets; these weights represent the amount of likelihood that can be associated to the focal sets but to no proper subset of them (*i.e.* portions of these weights may move freely from one element of the focal set to another) [Baudrit et al. 2006].

The function m is not the fundamental measure of likelihood of a proposition (set) A; rather, there are two measures of likelihood, called Belief and Plausibility, that are obtained from m as [Baudrit et al. 2006]: likelihood measures

$$Bel(A) = \sum_{E \subseteq A} m(E) \tag{1.1}$$

$$Pl(A) = \sum_{E \cap A \neq 0} m(E) \tag{1.2}$$

More intuitively, the *belief* in a proposition (set) A is quantified as the sum of the probability masses assigned to all sets enclosed by it; hence, it is a lower bound representing the amount of belief that directly supports the proposition at least in part. The *plausibility* of event A is, instead, the sum of the probability masses assigned to all sets whose intersection with the proposition is not empty; hence, it is an upper bound on the possibility that the proposition could be verified, *i.e.*, it measures the fact that the proposition could possibly be true "up to that value" because there is only so much evidence that contradicts it [Baraldi and Zio 2010].

1.2 Possibility theory

In possibility theory, uncertainty is represented using a *possibility function* $\pi_Y(y)$. For each y in a set S, $\pi_Y(y)$ expresses the degree of possibility of y. When $\pi_Y(y) = 0$ for some y, it means that the outcome y is considered an impossible situation. When $\pi_Y(y) = 1$ for some y, it means that the outcome y is possible, *i.e.*, is just unsurprising, normal, usual **[Dubois 2006]**. This is a much weaker statement than when probability is 1.

representing partial knowledge

In the possibilistic framework, extreme forms of partial knowledge can be expressed, such as:

- ▷ Complete knowledge: for some state s_0, $\pi(s_0) = 1$ and $\pi(s) = 0$ for other states s (only s_0 is possible)
- ▷ Complete ignorance: $\pi(s) = 1 \forall s \in S$ (all states are totally possible)

The possibility function $\pi_Y(y)$ gives rise to probability bounds, upper and lower probabilities, referred to as necessity and possibility measures (N_Y, Π_Y). The possibility of a set A, $\Pi_Y(A)$, is defined by

$$\Pi_Y(A) = \sup_{y \in A} \{\pi_Y(y)\} \tag{1.3}$$

and the necessity measure $N_Y(A)$ is defined by

$$N_Y(A) = 1 - \Pi_Y(\overline{A}) = 1 - \sup_{y \notin A} \{\pi_Y(y)\} \tag{1.4}$$

where $\overline{A}$ represents the complement of A. Let $\mathcal{P}(\pi_Y)$ be a family of probability distributions such that for all sets $A, N_Y(A) \leq P(A) \leq \Pi_Y(A)$. Then,

$$N_Y(A) = \inf P(A) \tag{1.5}$$

$$\Pi_Y(A) = \sup P(A) \tag{1.6}$$

where *inf* and *sup* are with respect to all probability measures in $\mathcal{P}$. Hence the necessity measure is interpreted as a lower level for the probability and the possibility measure is interpreted as an upper limit. Referring to subjective probabilities, the bounds reflect that the analyst is not able or willing to precisely assign his/her probability, and the bounds are the best he/she can do given the information available; in other words, he or she can only describe a subset of $\mathcal{P}$ which contains his/her probability **[Dubois and Prade 1988]**.

A typical example of possibilistic representation is the following **[Anoop and Rao 2008; Baraldi and Zio 2008a]**. We consider an epistemically-uncertain parameter y; based on its definition we know that the parameter can take values in the range [4, 6] and the most likely value is 5: to represent this information a triangular possibility distribution on the interval [4, 6] is used, with maximum value at 5, see figure 1.1.

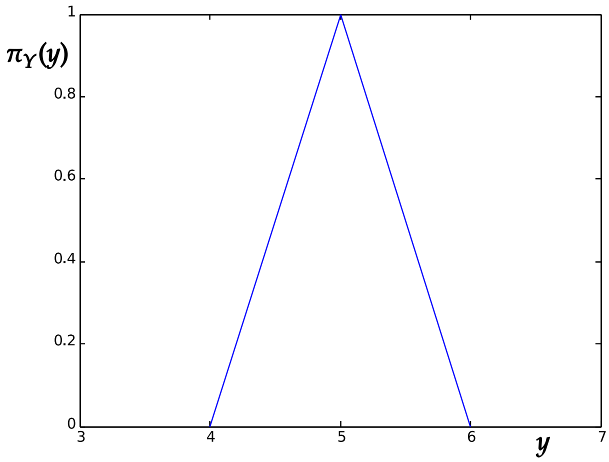

Figure 1.1 – *Possibility function for a parameter Y which is epistemically-uncertain on the interval [4,6], with maximum value at 5, representing an expert's qualitative assertion "the value is between 4 and 6, with the most likely value being 5"*

2

Uncertainty setting

Let us consider a model $\underline{\mathbf{Z}} = g(\underline{\mathbf{Y}})$, where $\underline{\mathbf{Z}} = (Z_1, Z_2, ..., Z_O)$ is the vector containing the O output variables of interest, and $g(\cdot)$ is a function that models how $\underline{\mathbf{Z}}$ depends on the k uncertain variables $Y_j, j = 1...k$, of vector $\underline{\mathbf{Y}}$; the uncertainty on these variables is characterized by known probability distributions $F_{Y_j}(y_j; \theta_j), j = 1...k$, where $\theta_j = \{\theta_{j,1}, ..., \theta_{j,M}\}$ are the vectors containing the hyper-parameters[1] of the corresponding probability distributions. These parameters are also uncertain and the information to characterize this uncertainty is drawn from an expert. This framework of analysis, where the aleatory and epistemic components of the uncertainty are separated into two hierarchical levels is often referred to as a 'level 2' approach or setting **[Limbourg and de Rocquigny 2010]** (*cf.* figure 2.1).

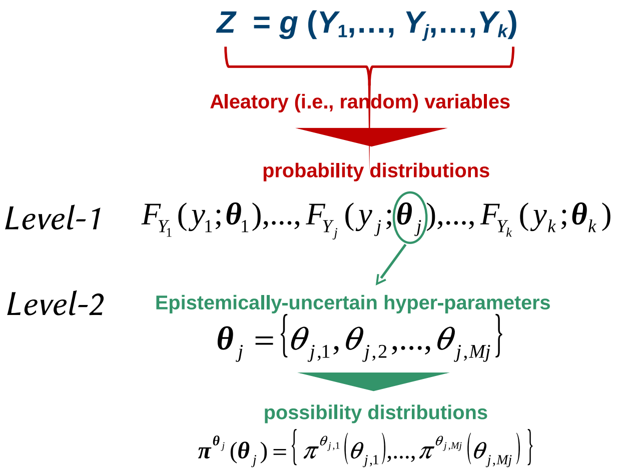

Figure 2.1 – *The "level-2" approach*

As mentioned, information is elicited from an expert for estimating the parameters $\theta_j, j = 1...k$. The associated uncertainty is represented within the framework of possibility theory, and propagated by means of the method based on the concept of FRVs[2]. For the sake of clarity of illustration, the treatment of uncertainty is described by ways of a simple case study concerning a non-repairable component whose state can only be either working or failed and whose Time To Failure (TTF) is exponentially distributed with uncertain failure rate λ. The mission time is T (taken equal to 10^5 hours in the numerical case study). Hence, in this reference example there are $k = 1$ uncertain variables, *i.e.* $\underline{Y} = (Y_1) = (TTF)$, described by the Cumulative Distribution Function (CDF) $F_{TTF}(ttf; \lambda) = 1 - e^{-\lambda ttf}$, with $M_1 = 1$ uncertain parameter $\theta_1 = \{\lambda\}$. The output vector $\underline{\mathbf{Z}}$ contains only one variable: the portion D of the

[1] In Bayesian statistics, a hyperparameter is a parameter of a prior distribution. The term is used to distinguish them from parameters of the model for the underlying system under analysis.

[2] FRV: Fuzzy Random Variable. A fuzzy random variable associates a fuzzy set to each possible result of a random experiment.

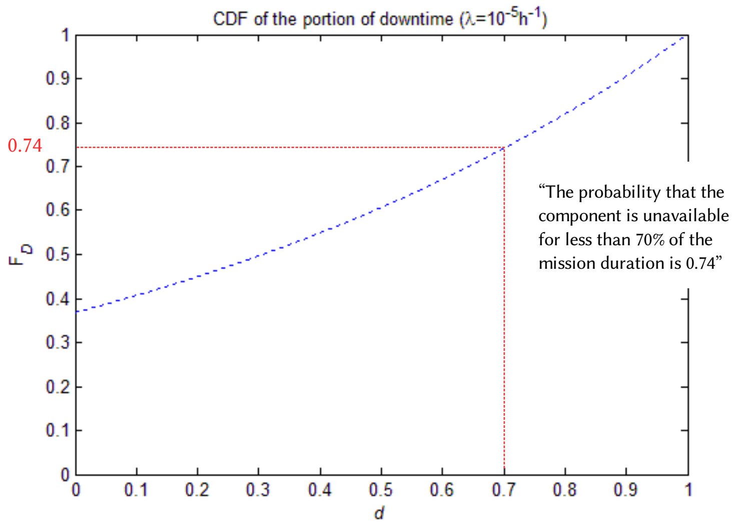

Figure 2.2 – *CDF of D, the proportion of mission time when the component is in a down state, for* $\lambda = 10^{-5}h^{-1}$

mission time in which the component is in a down state, *i.e.*, unavailable. The function g that links TTF to D is given by:

$$D = g(TTF) = \begin{cases} \frac{T-TTF}{T} & \text{if } TTF \leq T \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

Then, D is also a random variable, because it is a function of the random variable TTF. The range of variability of D is the interval [0,1], and its distribution, for a given value of the failure rate λ, is:

$$F_D(d|\lambda) = P(D \leq d|\lambda) = P\left(\frac{T - TTF}{T} \leq d|\lambda\right) = P\left(TTF \geq T(1-d)|\lambda\right) = e^{-\lambda T(1-d)} \tag{2.2}$$

where d represents the generic value taken by the variable D.

Figure 2.2 shows the shape of this function for a value of the failure rate $\lambda = 10^{-5}h^{-1}$. Notice that $F_D(0)$ (*i.e.* the probability that the component is always available during the mission time) is equal to $e^{-\lambda T}$, *i.e.* the probability that the component fails after T.

2.1 Information elicited from experts

nested intervals with increasing confidence level

Within the possibility theory framework, for a generic uncertain parameter θ, an expert is asked to provide a set of n nested intervals A_i, $i = 1...n$, $(A_1 \subseteq A_2 \subseteq \cdots \subseteq A_n)$, which are believed to contain the true value of θ with a positive confidence level q_i; this latter can be conveniently interpreted as the smallest (subjective) probability that the true value of the parameter θ lies within A_i (*i.e.*, $P(\upsilon \in A_i) > q_i$). Alternatively, the interval A_i can be seen as the smallest one whose probability of including the true value of θ is at least q_i [Sandri et al. 1995], for any $i = 1...n$. From the expert's point of view, q_i is the portion of cases where $\theta \in A_i$ from his/her experience [Sandri et al. 1995]. To sum up, the expert provides a weighted family $\{(A_1, q_1), (A_2, q_2), ..., (A_n, q_n)\}$ (see figure 2.3 for an example). Notice also that the value of the largest confidence level q_n may be smaller than 1, *i.e.* $q_n = 1 - \varepsilon, \varepsilon > 0$; this is equivalent to admitting that even the widest, safest interval contains some residual uncertainty (ε), *i.e.* it is assumed that the expert is not absolutely sure about his judgment [Sandri et al. 1995].

Finally, the inequalities $q_1 \leq q_2 \leq \cdots \leq q_n$ hold, due to the fact that q_i of the interval A_i is necessarily smaller than q_{i+1} associated to A_{i+1}, if $A_i \subseteq A_{i+1}$, for any $i = 1...n-1$.

With reference to the simple case study of the exponential, non-repairable, binary component, let us suppose that the expert characterizes his/her knowledge about the value of the failure rate λ with the information summarized in table 2.1.

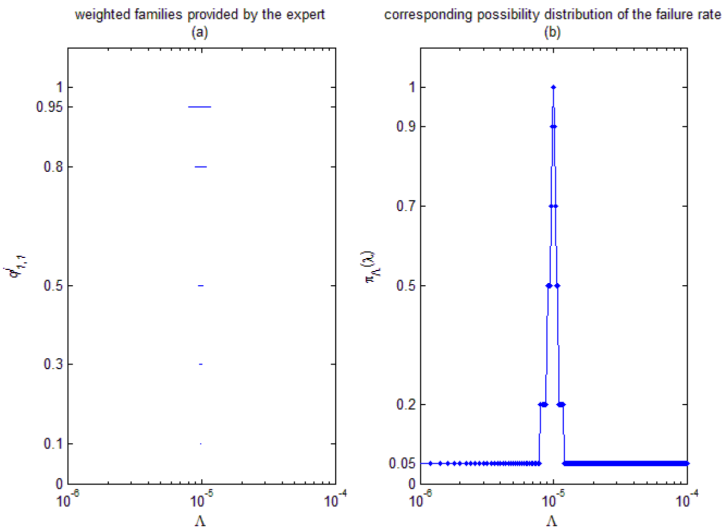

Figure 2.3 – *Representation of the weighted families for the failure rate λ provided by the expert for the exponential, non-repairable, binary component (left) and the corresponding possibility distribution, built according to the procedure proposed in* [Baudrit et al. 2006] *(right). Note the logarithmic scale on the horizontal axes.*

	Degree of certainty									
	$q_1 = 0.1$		$q_2 = 0.3$		$q_3 = 0.5$		$q_4 = 0.7$		$q_5 = 0.95$	
	min	max	min	max	min	max	min	max	min	max
λ [h⁻¹]	$9.9 \cdot 10^{-6}$	$1.01 \cdot 10^{-5}$	$9.7 \cdot 10^{-6}$	$1.03 \cdot 10^{-5}$	$9.5 \cdot 10^{-6}$	$1.05 \cdot 10^{-5}$	$9 \cdot 10^{-6}$	$1.1 \cdot 10^{-5}$	$8 \cdot 10^{-6}$	$1.2 \cdot 10^{-5}$

Table 2.1 – *Information elicited from the expert for the case study concerning the exponential, non-repairable, binary component*

The Universe of Discourse (UoD), *i.e.*, the interval of all possible values of the failure rate is [0, ∞[, where the lower bound (0) corresponds to an infallible component, whereas an infinite failure rate corresponds to a component that fails at $t = 0^+$. From table 2.1, it appears that the expert provides the interval A_1 that is believed to normally, unsurprisingly contain the true failure rate value, with confidence level $q_1 = 0.1$, which represents the portion of cases where $\lambda \in A_1$ from the expert's experience. The interval $A_1 = [9.9 \cdot 10^{-6} h^{-1}, 1.01 \cdot 10^{-5} h^{-1}]$ is 'unsurprising' in the sense that any interval A_1^* of the same length as A_1 would have been associated to an equal or smaller frequency of occurrence of the event $\lambda \in A_1^*$. Obviously, the expert cannot be less confident that the true value of the failure rate belongs to intervals that include A_1; thus, larger intervals are associated with larger confidence levels. In particular, $A_5 = [8 \cdot 10^{-6} h^{-1}, 1.2 \cdot 10^{-5} h^{-1}]$ is the interval which leaves an $\varepsilon = 0.05$ probability of not including the true value of λ.

Figure 2.3 (left) reports the set of intervals provided by the expert, and corresponding confidence levels (degrees of certainty). For visualization, both figure 2.3 (left) and (right) report only the interval $[10^{-6} h^{-1}, 10^{-4} h^{-1}]$, instead of the entire UoD, and the abscissa axes are logarithmically scaled.

More generally, in an uncertainty setting with k variables, an expert is asked to provide for every $j = 1 \ldots k$ and $p = 1 \ldots M_j$, a set of $n_{j,p}$ nested intervals $A_i^{j,p} \left(A_1^{j,p} \subseteq A_2^{j,p} \subseteq \cdots \subseteq A_{n_{j,p}}^{j,p} \right)$, $i = 1 \ldots n_{j,p}$, which are believed to contain the true value of the p–th parameter of the j-th random variable $\theta_{j,p}$ with a positive confidence level $q_i^{j,p}$.

2.2 Uncertainty representation

In this work, the uncertainty on the information elicited from the experts is represented using possibility theory (see chapter 1).

In our case, a possibility distribution is directly built from the weighted families $\{(A_1, q_1), (A_2, q_2), \ldots, (A_n, q_n)\}$ provided by the expert, according to the procedure proposed in **[Sandri et al. 1995]** and whose steps are here briefly recalled, for convenience.

▷ First of all, it is postulated that the necessity measure, $N(A_i)$, *i.e.*, the lower probability that the true value of θ is in the interval A_i, is equal to the confidence level q_i defined by the expert. Thus, the inequality $P(\upsilon \in A_i) \geq N(A_i) = q_i$ holds, for any $i \in 1 \ldots n$.

▷ Then, since there are infinite possibility distributions $\pi_\theta(\upsilon)$ that obey the constraint $q_i = N(A_i)$, it has been decided to choose the one which maximizes the degree of possibility $\pi_\theta(\upsilon)$ for all values υ. The solution is unique and is **[Dubois 2006]**:

$$\pi_\theta(\upsilon) = \begin{cases} 1 & \text{if } \upsilon \in A_1 \\ \min\limits_{i:\theta \notin A_i} (1 - q_i) & \textit{otherwise} \end{cases} \tag{2.3}$$

In particular, it is possible to show that this is the least specific possibility distribution with respect to the available data, *i.e.*, any other possibility distribution π_θ^1 obeying the constraints $q_i = N(A_i)$ is such that $\pi_\theta^1 \leq \pi_\theta$ **[Dubois 2006]**.

With reference to the case of the exponential, non-repairable, binary component, the possibility distribution π_A of the failure rate λ associated to the weighted family of table 2.1 and built according to the procedure depicted above, is reported in figure 2.3 (right). To verify that this distribution obeys the constraints $q_i = N(A_i)$ for $i \in 1 \ldots 5$, let us consider, for example, the first interval A_1; then, $N(A_1) = 1 - \Pi(\text{not } A_1) = 1 - \sup\limits_{\upsilon \notin A_1} \{\pi_\theta(\upsilon)\} = 1 - 0.9 = 0.1 = q_1$. Notice also the residual uncertainty $\varepsilon = 0.05$ associated to the points of the UoD external to A_5.

2.3 Uncertainty propagation

The uncertainty in the parameters of the model needs to be propagated to assess the uncertainty on the outputs. To this aim, we exploit the concept of FRVs within the methodology proposed in [Baudrit et al. 2008]. FRVs can be intuitively conceptualized as random variables whose values are not real numbers, but fuzzy numbers, since there is a vague perception of their true values, which are crisp but unobservable [Shapiro 2009]. In other words, a FRV is a generalization of a random variable or a fuzzy variable.

The operative steps of the uncertainty propagation procedure are reported in the following with reference to the case of the exponential, non-repairable, binary component. Since this case is characterized by a single uncertain variable ($k = 1$), we will always omit in the notations the subscript 1 referring to the uncertain variable.

1. For each uncertain variable $Y_j, j = 1...k$, sample a vector $\{\mathbf{u}_j^{\omega}\}, \omega = 1...N_T$ made of N_T uniform random numbers[3] in [0,1[; for example in our case, since $k = 1$, we need a vector of random numbers $\{u^{\omega}\}, \omega = 1...N_T$. In particular, let us assume that the first sampled value $u^1 = 0.65$.
2. Select a value of α_i on [0,1] and take as intervals of possible values the cuts $[\underline{\boldsymbol{\theta}}_j, \overline{\boldsymbol{\theta}}_j]_{\alpha_i} = \left\{[\underline{\theta}_{j,1}, \overline{\theta}_{j,1}]_{\alpha_i}, ..., [\underline{\theta}_{j,M_j}, \overline{\theta}_{j,M_j}]_{\alpha_i}\right\}$ corresponding to the possibility distributions of the parameters $\boldsymbol{\theta}_j = \{\theta_{j,1}, ..., \theta_{j,M_j}\}$, of the variables $Y_j, j = 1...k$; in our case, let us start from $\alpha_i = 1$: the interval of possible values for the parameter λ is $[9.9 \cdot 10^{-6} h^{-1}, 1.01 \cdot 10^{-5} h^{-1}]$ (see figure 2.3 (right)).
3. Identify the set of random intervals $[\underline{y}_j^{\omega}, \overline{y}_j^{\omega}]_{\alpha_i}$ of the variables $Y_j, j = 1...k$, corresponding to the random vector $\left\{u_1^{\omega}, ..., u_j^{\omega}, ..., u_k^{\omega}\right\}$, using the α_i-cut $[\underline{\boldsymbol{\theta}}_j, \overline{\boldsymbol{\theta}}_j]_{\alpha_i} = \{[\underline{\theta}_{j,1}, \overline{\theta}_{j,1}]_{\alpha_i}, ..., [\underline{\theta}_{j,M_j}, \overline{\theta}_{j,M_j}]_{\alpha_i}\}$ found at step 2). In particular, the ω-th random interval of the j-th variable, $\left[\underline{y}_j^{\omega}, \overline{y}_j^{\omega}\right]_{\alpha_i} = \left[\inf_{\theta_j \in \left[\underline{\theta}_j, \overline{\theta}_j\right]_{\alpha_i}} F_{Y_j}^{-1}(u_j^{\omega}; \boldsymbol{\theta}_j), \sup_{\theta_j \in [\underline{\theta}_j, \overline{\theta}_j]_{\alpha_i}} F_{Y_j}^{-1}(u_j^{\omega}; \boldsymbol{\theta}_j)\right]$, where $F_{Y_j}^{-1}(u_j^{\omega}; \boldsymbol{\theta}_j)$ is the quasi-inverse function of the CDF $F_{Y_j}(y_j; \boldsymbol{\theta}_j)$ of the random variable Y_j, for any value of the vector $\boldsymbol{\theta}_j$ [4]. This procedure can be regarded as an extension of the Monte Carlo (MC) sampling method, modified to take into account the fact that the parameters of the CDFs are fuzzy-uncertain in their UoDs: each sample from the uniform distribution is associated to an interval of values, instead of a single value (figure 2.4), so that different CDFs are obtained from the sampling, and lower and upper bounding CDFs can be identified.
   In the reference case study, the interval associated to the sample $u^1 = 0.65$ and $\alpha_i = 1$ is $[\underline{ttf}, \overline{ttf}]_1 = [1.63 \cdot 10^5 h, 1.66 \cdot 10^5 h]$ (figure 2.4 (left)). This is obtained by considering the two extremes of the interval of the uncertain parameter λ equal to $[\underline{\theta}, \overline{\theta}]_1 = [9.9 \cdot 10^{-6} h^{-1}, 1.01 \cdot 10^{-5} h^{-1}]$, which define the upper and lower exponential distributions, $1 - e^{-\overline{\theta} t}$ and $1 - e^{-\underline{\theta} t}$, respectively. Then, these functions are inverted to find the interval $[\underline{ttf}, \overline{ttf}]_1$, which is given by:

$$[\underline{ttf}, \overline{ttf}]_1 = \left[\frac{-ln(1 - u^1)}{\overline{\theta}}, \frac{-ln(1 - u^1)}{\underline{\theta}}\right]$$

   Notice that in this particular case, the interval $[\underline{ttf}, \overline{ttf}]_1$ is trivially obtained, since the inverse function of the exponential distribution is known. In general, it may be difficult to find the analytical expression of the minimum and maximum values of the inverse function $F_{F_j}^{-1}(U; \boldsymbol{\theta}_j)$, especially if it depends on a large number of parameters (*e.g.*, $M_j > 4$). In these cases, one has to devise efficient methods for identifying the minimum and maximum values of the random variable corresponding to the different combinations of the uncertain parameters $[\underline{\boldsymbol{\theta}}_j, \overline{\boldsymbol{\theta}}_j]_{\alpha_i} = \left\{[\underline{\theta}_{j,1}, \overline{\theta}_{j,1}]_{\alpha_i}, ..., [\underline{\theta}_{j,M_j}, \overline{\theta}_{j,M_j}]_{\alpha_i}\right\}$.

[3] N_T is the sample size of the Monte Carlo procedure. A larger simulation time T will require a larger N_T.

[4] If U is a random variable uniformly distributed on [0,1[, then $F_{Y_j}^{-1}(u_j^{\omega}; \boldsymbol{\theta}_j)$ has CDF $F_{Y_j}(u_j^{\omega}; \boldsymbol{\theta}_j)$.

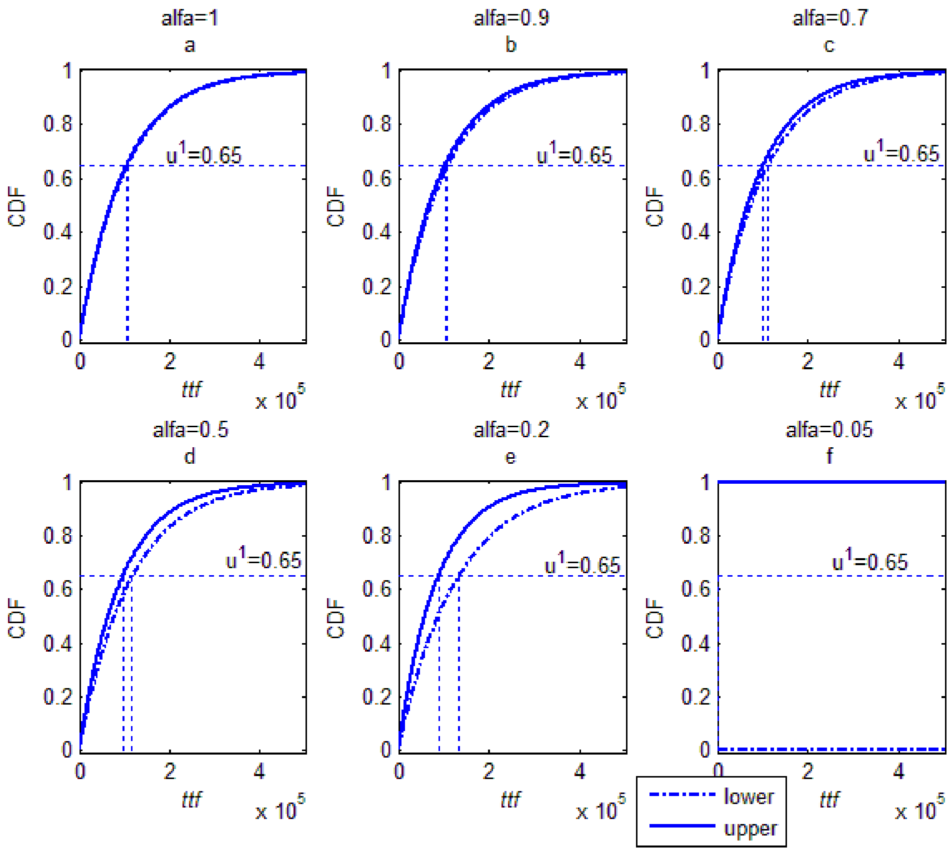

Figure 2.4 – *Lower and upper CDFs corresponding to different values of* α_i*, and the intervals associated by the quasi-inverse functions to* $u^1 = 0.65$

4. For every output variable Z_o, $o = 1...O$, calculate the smallest and largest values of g (denoted by $\underline{g}^{Z_O}_{\alpha_i}(\omega)$ and $\overline{g}^{Z_O}_{\alpha_i}(\omega)$, respectively), within the intervals $[\underline{y}^{\omega}_j, \overline{y}^{\omega}_j]_{\alpha_i}$, $j = 1...k$, of the variables:

$$\underline{g}^{Z_O}_{\alpha_i}(\omega) = \inf_{j, y_j \in [\underline{y}^i_j, \overline{y}^i_j]_{\alpha_i}} g(y_1, ..., y_j, ..., y_k) \tag{2.4}$$

$$\overline{g}^{Z_O}_{\alpha_i}(\omega) = \sup_{j, y_j \in [\underline{y}^i_j, \overline{y}^i_j]_{\alpha_i}} g(y_1, ..., y_j, ..., y_k) \tag{2.5}$$

for $o = 1, ..., O$, and consider the interval:

$$\Gamma^{z_O}_{\alpha_i}(\omega) = \left[\underline{g}^{Z_O}_{\alpha_i}(\omega), \overline{g}^{Z_O}_{\alpha_i}(\omega)\right] \tag{2.6}$$

In the case of the exponential, non-repairable, binary component, the minimum and maximum values of the TTF found in the previous step (*i.e.*, $1.63 \cdot 10^5$ h and $1.66 \cdot 10^5$ h, respectively) are both larger than the mission time $T = 10^5 h$, and thus the corresponding values of D are zero (equation (2.1)).

5. Return to step 2) and repeat steps 3) and 4) for another α-cut. For the exponential, non-repairable, binary component, the intervals $[\underline{ttf}^1_1, \overline{ttf}^1_1]_{\alpha_i}$ corresponding to different values of α_i are reported in figure 2.4. For example in the case of $\alpha_i = 0.5$, $[\underline{ttf}^1_1, \overline{ttf}^1_1]_{0.5} = [9.54 \cdot 10^4 h, 1.16 \cdot 10^5 h]$, whereas for $\alpha_i = 0.05$ the interval $[\underline{ttf}^1_1, \overline{ttf}^1_1]_{0.05} = [0, \infty[$.

6. The FRV corresponding to the ω-th realization is computed as:

$$\pi_{Z_o(\omega)}(z_o) = \sup[\alpha_i \in [0,1] | z_o \in \Gamma^{Z_o}_{\alpha_i}(\omega)] \tag{2.7}$$

The FRV that describes the portion of the component downtime associated to the first sample is shown in figure 2.5: since $\pi_{D(1)}(0) = 1$ it is fully plausible that the component is available for the overall mission time, whereas, since $\pi_{D(1)}(1) = 0.05$ it is not impossible that the component is unavailable for the entire mission time. Furthermore, according to the probabilistic interpretation of the possibility distribution, it is possible to observe that the probability that the component is fully available during its mission time is between 0.5 and 1, and the probability that the portion of downtimes is larger than 0.07 is between 0 and 0.2. Notice that the FRV of figure 2.5 is consistent with the intervals represented in figure 2.4. In fact, only the intervals corresponding to $\alpha_i \leq 0.5$ contain the value $T = 10^5 h$: this means that only for these values of α_i the component may experience a failure before T, which entails its unavailability for the remaining part of the mission time.

7. Repeat steps 1)-6) for a new realization of the random variables, until $\omega = N_T$.

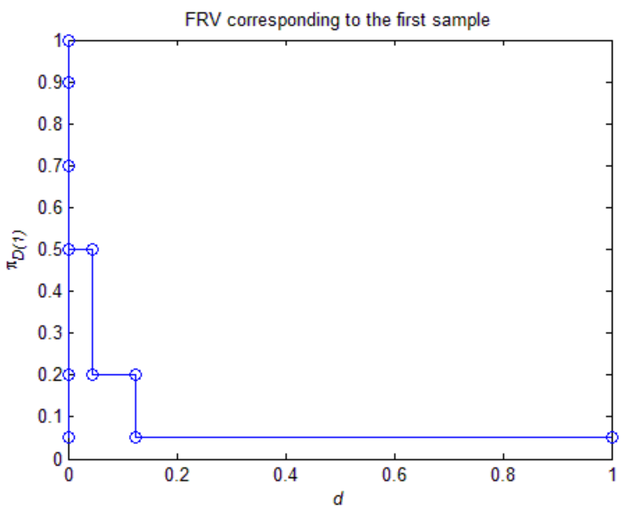

Figure 2.5 – *Fuzzy Random Variable corresponding to the sample* $u^1 = 0.65$

8. Compute the Plausibility and Belief distributions for Z_o, $o = 1 \ldots O$ by:

$$Pl\left(Z_o \in]-\infty, z_o]\right) = \sum_{\omega=1}^{N_T} \frac{1}{N_T} \cdot \sup_{z_o \in]-\infty, z_o]} \pi_{Z_o(\omega)}(z_o) \tag{2.8}$$

$$Bel\left(Z_o \in]-\infty, z_o]\right) = \sum_{\omega=1}^{N_T} \frac{1}{N_T} \cdot \inf_{z_o \notin]-\infty, z_o]} \left(1 - \pi_{Z_o(\omega)}(z_o)\right) \tag{2.9}$$

where $1/N_T$ is the probability assigned to the ω-th FRV, for any ω. In particular, equation (2.9) is derived from the interpretation of the FRVs under the setting of random sets **[Baudrit et al. 2006]**.

The Plausibility and Belief distributions of D (*i.e.*, the upper and lower bounds, respectively, of the probability distributions of the portion of the mission time T in which the exponential, non-repairable, binary component is in a fault state) are reported in figure 2.6; for comparison, the CDF (see equation (2.9)) is also provided, which lies between the Plausibility and Belief distributions.

A comment seems in order about the requirement that the uncertainties on all the input parameters must be described by the same expert, which is mandatory for applying this procedure. This constraint comes from the application of the extension principle in equation (2.7), which introduces a strong dependence between the information sources supplying the input possibility distributions. Indeed, the same confidence level for all the input variables is chosen to build the α-cuts of the output variables; this suggests that if the expert source informing on one parameter is rather precise or gives the same mean values to the confidence levels, then the one informing on another parameter must also be precise, *i.e.*, to ensure this, it must be the same source. Further research effort should be spent in order to verify whether the procedure here illustrated can be interpreted as a conservative counterpart to the calculus of probabilistic variables under stochastic independence, due to the dependence between the choice of confidence levels.

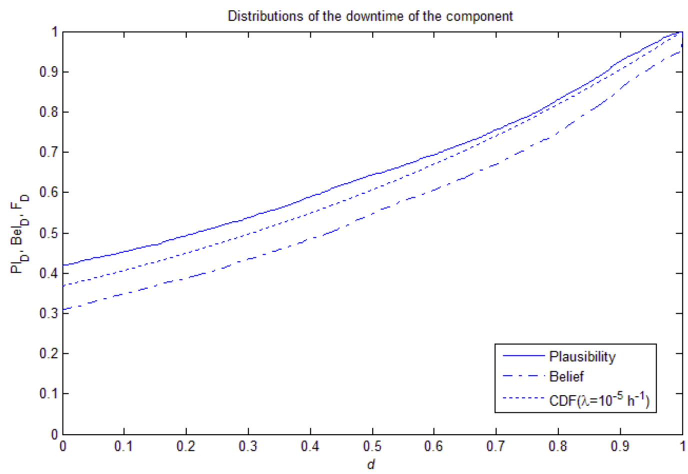

Figure 2.6 – *Plausibility, Belief and Cumulative distributions of the portion of mission time in which the component is in a fault state*

3

Case study

The present case study is taken from **[Zille et al. 2009]** and regards the degradation and maintenance of a check valve of a turbo-pump lubricating system in a Nuclear Power Plant. The degradation modeling is based on information collected from dependability analyses (*e.g.* FMECA) or directly from experts. This leads to the identification of one principal degradation mechanism, *i.e.*, fatigue, and a single failure mode, *i.e.* rupture. A Condition-Based Maintenance (CBM) policy is applied to this component on a time horizon $T = 10^4 h$. The performance of the maintenance policy is assessed in terms of cost and component unavailability.

Condition-based maintenance (CBM)

DEFINITION

Condition-based maintenance designates a policy of performing maintenance only when the need arises. This maintenance is performed after one or more indicators show that equipment is going to fail or that equipment performance is deteriorating.

Compared with more traditional planned maintenance, CBM promises to improve system reliability, decrease maintenance costs and decrease the possibility for human error during maintenance operations. Its disadvantages are increased equipment costs (added cost of instrumentation and monitoring devices), unpredictable maintenance periods and extra complexity due to the monitoring equipment.

3.1 Degradation mechanism modeling

The fatigue phenomenon affecting the check valve is here modeled as a discrete-state, continuous-time stochastic process that evolves among the following three degradation levels (figure 3.1):

1. 'Good': a component in this state is new or almost new (no crack is detectable by maintenance operators);
2. 'Medium': if the component is in this degradation level, then it is convenient to replace it;
3. 'Bad': a component in this degradation state is very likely to experience a failure in few working hours.

The choice of describing the degradation process by means of a small number of levels, or degradation 'macro-states', is driven by industrial practice: experts usually adopt a discrete and qualitative classification of the degradation states based on qualitative interpretations of symptoms.

The probability density functions (pdfs) of the transition times are Weibull distributions, with scale parameters η_{ij} and shape parameters β_{ij} for the transitions from state i to state j ($i, j \in \{1, 2, 3\}$ and $i < j$). The Weibull distribution is commonly applied in fracture mechanics (*e.g.*, **[Wormsen and Härkegård 2004]**), especially under the weakest-link assumption **[Remy et al. 2010]**[1].

[1] Waloddi Weibull proposed his eponymous distribution in 1939 to model the variation in the relative tensile strength of various materials as a function of their size. Brittle fractures in a material under stress are caused by the presence of defects (presence of impurities introduced during casting, for example), which can be assumed to be uniformly distributed in the material. The Weibull distribution arises from the weakest-link model for brittle fractures, the

A further state, 'Failed', can be reached from every degradation state upon the occurrence of a shock event. The exponential distribution with constant failure rate λ_j describes the failure behaviour of the component while it is in state j, for every $j = 1, 2, 3$. The choice of assigning a constant failure rate to every degradation state is driven by industrial practice: experts are familiar with this setting and comfortable with providing information about the failure rate values.

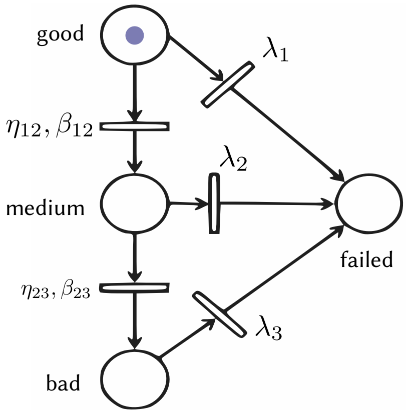

Figure 3.1 – *Degradation model of the check valve of a turbo-pump lubrication system*

3.2 Maintenance policy

The CBM policy applied to the system is composed by the following tasks:

- ▷ Inspections: these actions, which are the only scheduled actions, are aimed at detecting the degradation state of the component, and are considered to last 5h for a cost of 50€. For the sake of simplicity, the component is considered as new after the inspection.
- ▷ CBM actions: Preventive Maintenance (PM) actions which are dependent on the result of an inspection action. More precisely, if the component is found to be in state 'Good', no action is performed, whereas if the degradation state is 'Medium' or 'Bad', then the component is replaced and, consequently, the degradation state is taken back to 'Good'. Both these replacement actions are assumed to take 25h and cost 500€ each.
- ▷ Corrective Maintenance (CM) actions. The corrective action, performed after a component failure, is assumed to be the replacement of the component. Due to the fact that this event is unscheduled, this action brings an additional duration of 85h and an additional cost of 3500€, with respect to the replacement after an inspection, leading to a total duration of 100h and to a total cost of 4000€. In particular, the additional time may be caused by the supplementary time needed for performing the procedure of replacement after failure or to the time elapsed between the occurrence of the failure and the start of the replacement actions.

The Inspection Interval (*II*), which is the time span between two successive planned inspections, is the only decision variable considered in this case study; optimization is then directed to the search for the value of the *II* that minimizes the costs and maximizes the availability of the component.

notion that a volume of material breaks at its weakest point, or that the probability that a homogeneously stressed volume escapes brittle fracture is given by the probability that all the volume elements comprising it survive. The Weibull probability distribution is today widely used in the reliability field.

3.3 Analysis of the case study

The case study is firstly investigated in the unrealistic situation in which the values of the model parameters $\theta_{j,p}, j = 1...k$ and $p = 1...M_j$ are assumed to be exactly known (*i.e.*, there is no epistemic uncertainty). Table 3.1 reports the values of these parameters, which have been taken from [Zille et al. 2009].

Parameters	Nominal values
$\theta_{1,1} = \eta_{12}$	1861h
$\theta_{1,2} = \beta_{12}$	8
$\theta_{2,1} = \eta_{23}$	743h
$\theta_{2,2} = \beta_{23}$	8
$\theta_{3,1} = \lambda_1$	$10^{-6}h^{-1}$
$\theta_{4,1} = \lambda_2$	$10^{-4}h^{-1}$
$\theta_{5,1} = \lambda_3$	$10^{-2}h^{-1}$

Table 3.1 – *Parameters of the probability distributions for the degradation model shown in figure 3.1 (situation without epistemic uncertainty)*

Figure 3.2 shows the CDF of the portion of the mission time in which the component is unavailable. Two main steps in the CDF can be observed, which can be explained by analyzing the Monte Carlo simulation results, where almost 60% of the population experience one of the following two behaviors:

- ▷ The component never fails during the mission time, and thus is inspected 4 times (at t = 2000h, 4000h, 6000h and 8000h); in 3 of these 4 inspections the component is found in degradation states Medium or Bad (75h of downtime) and in the remaining one in degradation state Good (5h of downtime). Thus, the total downtime is 80h, which constitutes 0.8% of T. Components experiencing this life explain the CDF step at d = 0.008.
- ▷ The component never fails during the mission time, and when inspected is always found in degradation states Medium or Bad (100h of downtime). This behavior explains the CDF step in correspondence of d = 0.01.

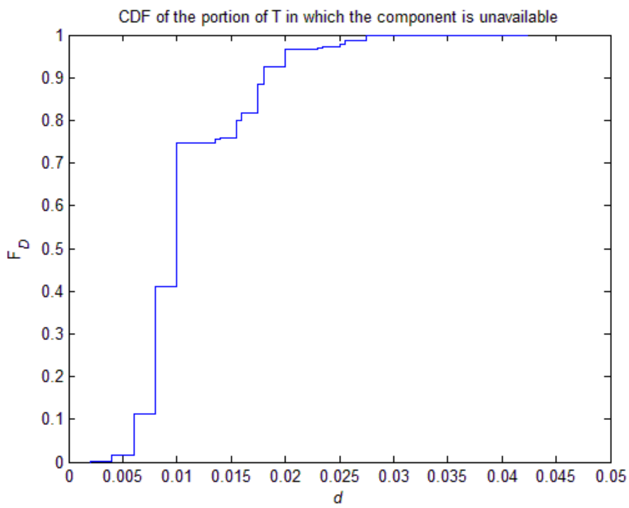

Figure 3.2 – *CDF of the portion of the time horizon in which the component is in a down state*

Notice that it is possible to lump together the information provided by the cumulative distribution of the portion of downtime into the mean value of the distribution, *i.e.*, the average unavailability over the mission time, which provides an useful and easily interpreted indicator of the component's expected state during the mission. The estimated average unavailability is 0.011, and the related 68.3% confidence interval is $[0.011 - 9.8 \cdot 10^{-8}, 0.011 + 9.8 \cdot 10^{-8}]$ (this confidence interval is related to the Monte Carlo modelling, and not to any epistemic uncertainty in the inputs).

3.4 Maintenance optimization

Figure 3.3 shows the estimated average unavailability of the component over the mission time (*i.e.*, the mean value of the component downtime over the entire mission time), with the related 68.3% confidence interval, for different values of the inspection interval *II*. The narrowness of the confidence intervals is due to the large number ($5 \cdot 10^4$) of Monte Carlo simulations performed in this case study; roughly speaking, the larger this number, the smaller the (confidence) interval that with a given probability (confidence level) contains the true value of the estimated variable. Thus, in the present case study the actual value of the average unavailability over the mission time is affected by a small amount of estimation error, which can be reduced by increasing the number of simulations.

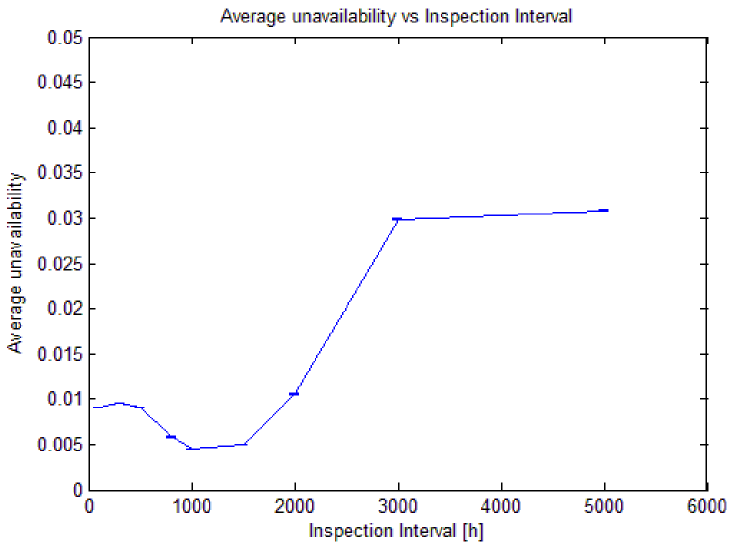

Figure 3.3 – *Average unavailability corresponding to different Inspection Intervals*

Initially, there is a decreasing behavior that reaches a minimum at *II*=1000h/1500h; after this point, the unavailability starts rapidly increasing. This is the result of two conflicting trends: on one side, more frequent inspections lead to a greater probability of finding the component in degradation states Medium and Bad: this prevents component failure and avoids the corresponding large time to repair. On the other side, frequent replacements are inefficient, since the component life is not completely exploited in this case. The minimum at *II*=1500h represents the optimal balance between these two tendencies.

Figure 3.4 shows that the maintenance costs associated with different choices of the *II* have a shape similar to that of mean unavailability. Thus, one may conclude that under the considered maintenance policy, the best *II* is between 1000h and 1500h with respect to both availability and cost objectives. On the other hand, both the mean unavailability and the maintenance cost remain small, with little variation, when the value of the *II* ranges in the interval [1000h, 2000h]. This relative flatness of both performance indicators (unavailability and cost) allows a certain freedom to choose the *II* within such range: other criteria (*e.g.*, opportunistic maintenance) not included in this analysis can be taken into account if the related advantages recover the small losses due to the increase of unavailability and cost, that would be incurred when moving away from the optimal value of 1500h.

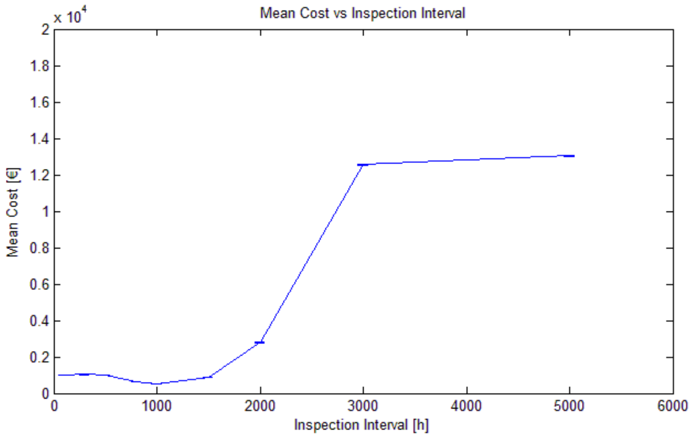

Figure 3.4 – *Mean cost corresponding to different Inspection Intervals*

3.5 Representation and propagation of the uncertainties

In this section, we apply the method illustrated in chapter 2 to the case study described above, when the parameters of the distributions that model the transitions of the component among the four states of figure 3.1 are ill-known and there is only one expert who estimates their values.

To sum up, the uncertainty situation is the following:

- ▷ There are $k = 5$ uncertain variables, which define the 5 transition times reported in table 3.2.
- ▷ The distributions associated with each variable are known, and depend on the set of the uncertain parameters $\theta_j, j = 1...5$ reported in table 3.2. In turn, there are N = 7 uncertain parameters, which are the shape and scale parameters of the two Weibull distributions and the failure rates pertaining to the three degradation levels (see table 3.2).

Uncertain variables	Uncertain parameters	Description
Y_1	$\theta_1 = \{\theta_{1,1}, \theta_{1,2}\}$	Transition time from degradation level 'Good' to 'Medium'
Y_2	$\theta_2 = \{\theta_{2,1}, \theta_{2,2}\}$	Transition time from degradation level 'Medium' to 'Bad'
Y_3	$\theta_3 = \{\theta_{3,1}\}$	Transition time from degradation level 'Good' to 'Failed'
Y_4	$\theta_4 = \{\theta_{4,1}\}$	Transition time from degradation level 'Medium' to 'Failed'
Y_5	$\theta_5 = \{\theta_{5,1}\}$	Transition time from degradation level 'Bad' to 'Failed'

Table 3.2 – *Tailoring the general model to the considered case study*

Notice that the simulation of a single Monte Carlo history (steps 1-5 of the procedure in § 2.3) requires that the model g encodes a number of random variables $k \gg 5$, since the history corresponding to a given sample of these 5 uncertain times in general do not cover the entire time horizon T. For example (figure 3.5 (a)), if the first transition is from state 1 to state 'Failed' and occurs at t=2000h, then the interval time between t=2000h+100h (*i.e.*, the time instant at the end of the repair action that starts after the failure) and T remains not investigated. This problem can be overcome by thinking of g as a function that depends on a number K of size-5 tuples (the 5 probabilistic variables), and not just on 5 variables. Obviously, the number K that allows to cover the entire mission time is also a random variable, since it depends on the sampled times, which produce histories of different lengths. However, this is not a problem in practice: the number K can be chosen such that it is reasonably sure that the sampled times simulate histories of duration larger than the time horizon T. Then, the analysis focuses only on the interval [0, T] (figure 3.5 (b)). Finally, the output vector Z is made up of the portion of

T in which the component is unavailable, and the cost associated to the maintenance policy to be assessed; thus $O = 2$.

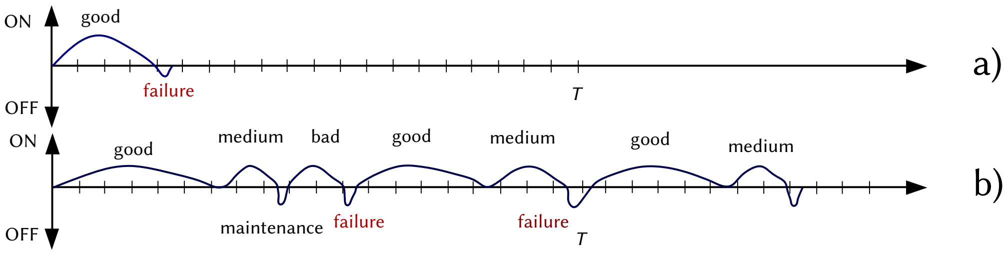

Figure 3.5 – *Two examples of simulated histories: the number of random variables does not suffice to cover the entire time horizon T (a); number K allows simulation of histories longer than T (b)*

3.6 Single expert information

In all generality, the difficulty in estimating the uncertain parameters of the model may heavily vary from one case to another; the weighted families $\left\{(A_1^{j,p}, q_1^{j,p}), (A_2^{j,p}, q_2^{j,p}), \dots, (A_{n_{j,p}}^{j,p}, q_{n_{j,p}}^{j,p})\right\}$ provided by the expert to represent his/her knowledge about the parameters are expected to reflect this difference.

The weighted families elicited from the single expert for this case study are reported in table 3.3. It is assumed that the expert is able to infer the information on the transition times between the different states, from the observations gathered during past component inspections. For example, let us suppose that a component is monitored every 100h, and that it was found in degradation state Medium at $t = 1800$h; if the component is found in degradation state Bad upon the next observation at $t = 1900$h, then the expert acquires the information that the transition from degradation state Medium to Bad occurred in the interval $]1800h, 1900h[$. This kind of information can be used to define the scale and shape parameters of the Weibull distributions representing these transitions. Notice, however, that the amount of uncertainty affecting the estimation of the scale and shape parameters is expected to be very different: the expert has a more refined knowledge on the scale parameter which can be seen as the time until which almost the 65% of the components have experienced a transition, than on the shape parameter which is only related to the slopes of the Weibull probability plots; these are expected to be difficult to estimate from the observations of the components' degradation states during the inspections.

Parameters		Confidence levels							
		$q_1^{j,p} = 0.1$		$q_2^{j,p} = 0.5$		$q_3^{j,p} = 0.95$		UoD	
		min	max	min	max	min	max	min	max
$\theta_{1,1}$	η_{12}	1843	1880	1815	1908	1720	2001	1700	2020
$\theta_{1,2}$	β_{12}					7.5	8.5	7	9
$\theta_{2,1}$	η_{23}	735	750	725	762	687	800	650	850
$\theta_{2,2}$	β_{23}					7.5	8.5	7	9
$\theta_{3,1}$	λ_1					$9 \cdot 10^{-7}$	$1.1 \cdot 10^{-6}$	$1 \cdot 10^{-7}$	$5 \cdot 10^{-6}$
$\theta_{4,1}$	λ_2					$9 \cdot 10^{-5}$	$1.1 \cdot 10^{-4}$	10^{-5}	$5 \cdot 10^{-4}$
$\theta_{5,1}$	λ_3					$0.9 \cdot 10^{-2}$	$1.1 \cdot 10^{-2}$	$0.85 \cdot 10^{-2}$	$1.15 \cdot 10^{-2}$

Table 3.3 – *Confidence levels and associated intervals*

With regards to the scale parameters (rows 4 and 6 of table 3.3), it has been assumed that the expert provides three nested intervals corresponding to the confidence levels $q_1^{j,p} = 0.1$, $q_2^{j,p} = 0.5$ and $q_3^{j,p} = 0.95$, $j = \{1, 2\}$ and $p = 1$, and the UoDs within which these parameters range.

With regards to the shape parameters (rows 5 and 7 of table 3.3), given the difficulty in their estimation, it has been assumed that the expert provides just the UoDs and the intervals corresponding to the confidence level $q_3^{j,p} = 0.95$. In particular, the UoDs, which contain the true values of the parameters with probability 1, are very large: the expert tends to reduce the sets of values that surely do not contain the true values of the scale parameters.

Finally, with reference to the failure rates, the estimation of the Mean Time To Failure (MTTF, *i.e.*, the inverse of the failure rate) of the components in a given degradation state may not be easy; in fact, failure from the first degradation state is usually a rare event, whose frequency is difficult to estimate even in a qualitative way, whereas the lack of precise knowledge of the time instants in which the components transition towards the other degradation states affects the evaluation of the mean times to failure associated to these states; that is, if the time instant since one has to start to count is unknown, then the resulting measure of the time to failure is biased, especially if the component is rarely inspected. Thus, a more vague description of the uncertainty is provided by the expert for these parameters of the model g. Namely, he/she gives just the intervals corresponding to the 0.95 confidence level and the UoD, as for the scale parameters of the Weibull distributions. The extreme points of these intervals are reported in rows 8-10, columns 7-10 of table 3.3.

On the other hand, the larger the number of the uncertain parameters, the larger the space in which the maxima and minima of the function g in equations (2.4) and (2.5) have to be searched for, and the larger the required computational time. In this regard, a sensitivity analysis can be performed preventively in order to identify which are the input parameters whose variations lead to smaller changes in the output value; this allows to neglect the uncertainty affecting these parameters while losing a small amount of information and considerably reducing the computational times.

In the present case study, the sensitivity analysis is performed by a local approach **[Zio 2009]**, *i.e.*, the uncertain parameters of the model are varied one by one within their UoD, while the other parameters take their nominal values. The results of the analysis are reported in table 3.4: the portion of T in which the component is unavailable is estimated in correspondence of the extreme values of the UoD of every uncertain parameter. For example, the estimation of D is 0.0142 in correspondence of the lower bound of the UoD of the scale parameter of the first Weibull distribution (1700h), whereas it is 0.0082 in correspondence of the upper bound (2020h). In particular, these values are reported with the related 68.3% confidence interval. The last column of the table reports Δ, *i.e.*, the differences between the average unavailability corresponding to the two limiting situations. These quantities give an estimation of the amount of output uncertainty (*i.e.*, the unavailability uncertainty) which is due to the variation of the model parameters. In practice, high values of Δ indicate the importance of properly considering the uncertainty in the parameters, whereas low values correspond to parameters whose uncertainty has no remarkable effect on the model output uncertainty.

In this case, the failure rate associated to the degradation state 'Bad' turns out to be the parameter which the model is less sensitive to. Then, the uncertainty affecting this parameter is not considered and the nominal value (table 3.1) is assigned to it.

Notice that the set of unavailability values reported in columns 2 and 3 of table 3.4 are not useful for representing the uncertainty on the unavailability, which must take into account not only the input parameter extreme values, but all the available information on the input parameter uncertainties, *i.e.*, the possibility distributions. Thus, the sensitivity analysis proposed here cannot replace the uncertainty representation and propagation tasks carried out in this work, but can be used to identify the input parameters whose uncertainty is most relevant.

Finally, notice that for every uncertain parameter and for any confidence level, the value considered in **[Zille et al. 2009]** is the middle point of the corresponding intervals provided by the expert.

Parameter	Minima	Maxima	Δ
η_{12}	$0.0142 \pm 1.2 \cdot 10^{-7}$	$0.0082 \pm 8.3 \cdot 10^{-8}$	$6.0 \cdot 10^{-3}$
β_{12}	$0.0116 \pm 1.1 \cdot 10^{-7}$	$0.0108 \pm 8.8 \cdot 10^{-8}$	$0.8 \cdot 10^{-3}$
η_{23}	$0.0119 \pm 1.0 \cdot 10^{-7}$	$0.0105 \pm 8.7 \cdot 10^{-8}$	$1.4 \cdot 10^{-3}$
β_{23}	$0.0112 \pm 9.9 \cdot 10^{-8}$	$0.0110 \pm 1.2 \cdot 10^{-7}$	$0.2 \cdot 10^{-3}$
λ_1	$0.0110 \pm 9.5 \cdot 10^{-8}$	$0.0114 \pm 9.6 \cdot 10^{-8}$	$0.4 \cdot 10^{-3}$
λ_2	$0.0103 \pm 8.2 \cdot 10^{-8}$	$0.0144 \pm 1.4 \cdot 10^{-7}$	$4.1 \cdot 10^{-3}$
λ_3	$0.0110 \pm 9.5 \cdot 10^{-8}$	$0.0111 \pm 8.8 \cdot 10^{-8}$	$0.1 \cdot 10^{-4}$

Table 3.4 – *Results of the local sensitivity analysis*

3.7 Possibilistic representation of the epistemic uncertainties

Figure 3.6 reports the possibility distributions of the uncertain parameters of the case study, corresponding to the weighted families of table 3.3. These are obtained by applying the procedure showed in § 2.2.

3.7.1 Uncertainty propagation

Figure 3.6 shows the results obtained by applying the FRVs-based method to the considered case study. The Plausibility and Belief distributions of the portion of the mission time in which the component is in a down state are quite distant; this shows that for some favorable combinations of the uncertain parameters the system results to be much more available than for other combinations of the uncertain parameters which lead to high portions of downtime. Notice also that, as expected, the Plausibility and Belief distributions bracket the CDF found in the case in which the uncertainty on the model parameters is not accounted for (§ 3.3).

lack of easily understood indicators

Notice that the results provided by the method discussed in this work are **difficult to interpret**. This is due to the fact that, unlike the case of no uncertainty, it is not possible to lump together the information provided by the method, *i.e.*, the Plausibility and Belief function, into indicators such as their mean values which are easy to interpret. This impossibility is due to the fact that the DSTE does not allow the definition of the mean value of an uncertain variable. However, in order to give an interpretation to the obtained results, one can focus on a given percentile of the belief and plausibility distributions. For example, the interval bounded by the values of the 95th percentile of the Plausibility and Belief distributions is [0.015, 0.026]; the extremes of this interval constitute the lower and upper bounds, respectively, of the 95th percentile of the portion of downtime in the mission time. In other words, this interval tells us that the 95% chance of the downtime of the component can be neither smaller than 1.5% nor larger than 2.6% of the mission time. Thus, if one is interested in the worst case, then one can assume that the 95th percentile of the downtime is 0.026, whereas in a more optimistic view, it can be valued at 0.015.

Figures 3.8 and 3.9 report the Plausibility and Belief distributions of the portion of downtime over the mission time and the total cost, respectively, corresponding to three different values of the II, *i.e.*, II=1000h, II=1500h and II=2000h. These results do not lend themselves to an easy interpretation and do not allow to make a decision in a simple way. Indeed, while it is easy to state that inspecting the component every 1000h is better than every 2000h, since these distributions do not overlap, answering the question 'which value of the II is best?' is not trivial, as the distributions corresponding to II=1500h and II=1000h overlap. This calls for devising a method in support of maintenance decision-makers, to help them get around these distributions. Notice also that the small amount of uncertainty on the values of both downtime and cost, when the component is inspected every 1000h, derives from the fact that the 'crowd' of the simulated Monte Carlo component histories (*i.e.*, the large number of components experiencing the same behavior) remains very compact in this case.

On the contrary, when the uncertainties affecting the parameters are not accounted for, the identification of the best value of II is more straightforward, since it is usually accepted to consider the mean value of the portion of mission time in which the component is faulty or the mean value of the cost as good indicators of the performance of the maintenance policy.

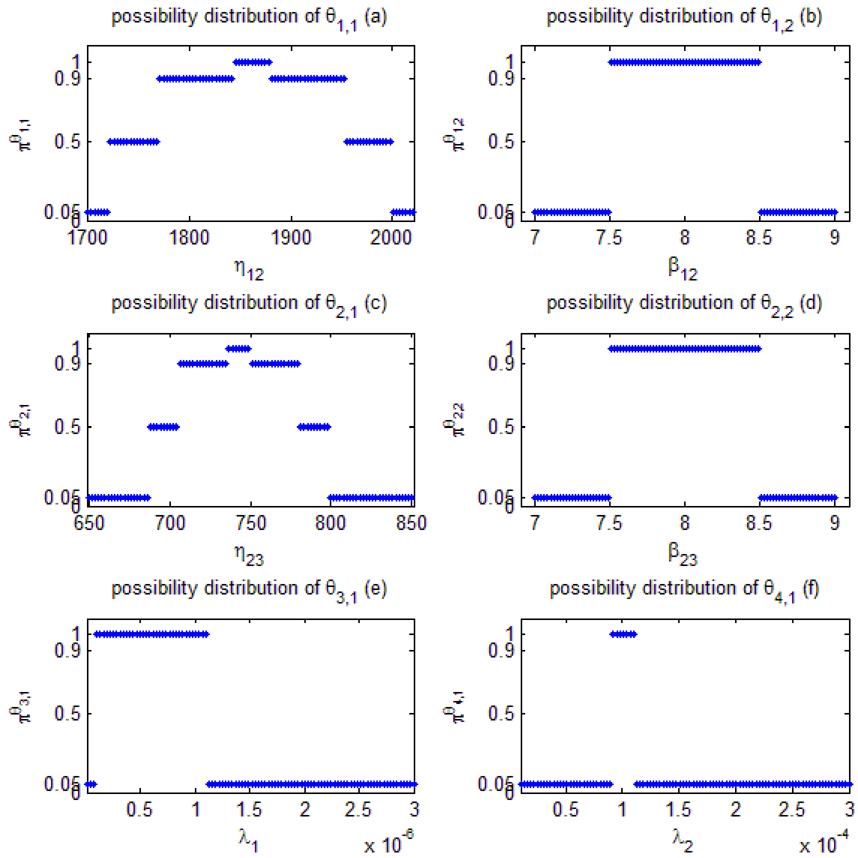

Figure 3.6 – *Possibility distributions of the uncertain parameters of the case study*

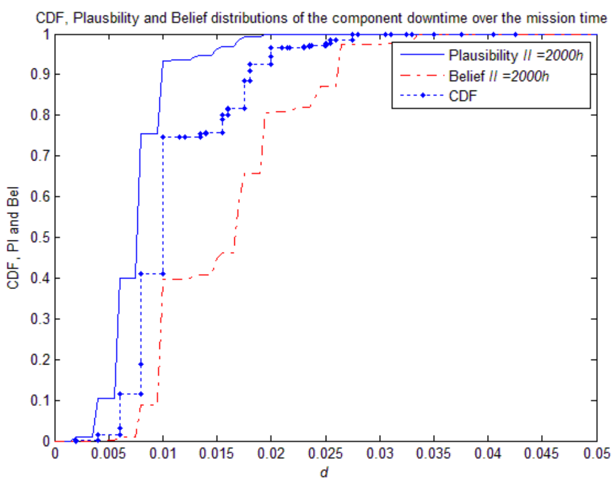

Figure 3.7 – *Plausibility and Belief distributions of the portion of mission time in which the component is unavailable, and the CDF corresponding to the case with no uncertainty*

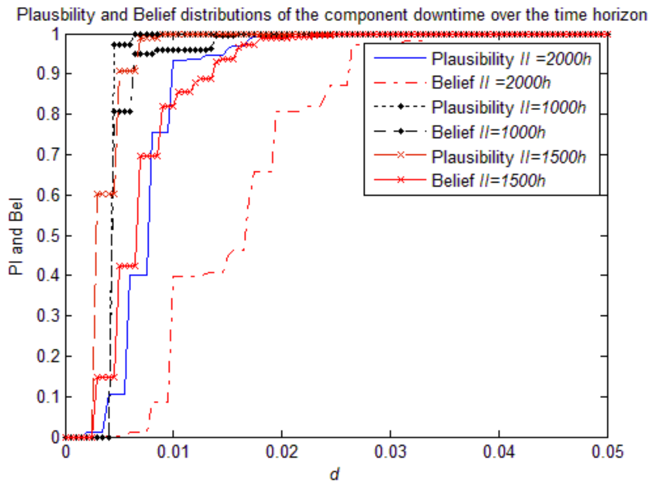

Figure 3.8 – *Plausibility and Belief distributions of the portion of time horizon in which the component is in a fault state, for different values of the control variable II*

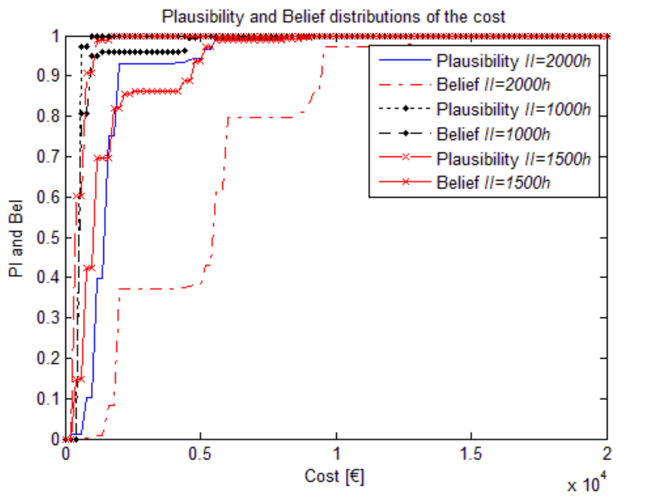

Figure 3.9 – *Plausibility and Belief distributions of the cost associated to the maintenance policy, for different values of the control variable II*

4

Conclusions

Uncertainty affects the parameters of the models of the behavior of components subject to a given maintenance policy. Incorrect treatment of such uncertainty may lead to serious bias of the outcome of the analysis, possibly non-conservatively. In practice, experts are often the only source of information on these parameters, and provide it in an ambiguous and qualitative form. Most commonly, all that is known is that a certain value belongs to a certain interval [Nicolai et al. 2009]. The representation of the uncertainty of this information in terms of probability distributions would force a set of assumptions, with introduction of biases and loss of generality. In this work, a methodology has been proposed based on the following steps:

1. Elicitation of the expert knowledge on the model parameters.
2. Representation of the uncertainty associated to the expert's judgment, avoiding introduction of unjustified, biasing assumptions. In this respect, notice that the choice of any probability distribution to represent the uncertainty in the expert's assignments would be absolutely arbitrary, if the expert is not able to provide this additional information.
3. Propagation of the uncertainty to the maintenance performance indicators.

The methodology has been applied to a case study concerning the degradation model of a check valve of a turbo-pump lubricating system in a nuclear power plant. The study has shown that neglecting uncertainty may drive the maintenance decision-maker towards incorrect conclusions. In this case, if the unavailability computation were performed without taking into account the uncertainty on the input parameters, the decision-maker would set the inspection intervals between maintenance actions to the value of II=1000h, whereas a proper consideration of the uncertainties through the use of FRV suggests that, on the basis of the available knowledge, this choice for the maintenance inspection interval is not better than other intervals such as II=1500h and II=2000h.

The main limitations of the methodology discussed in the present document are:

- ▷ It is necessary for a single expert to be knowledgeable, at least qualitatively, on all uncertain parameters and, furthermore, to be able to provide intervals of values for the uncertain parameters with associated confidence levels: this may be very difficult in practice. However, the FRVs-based method can be also applied when the expert provides just one interval of possible values per parameter, thus avoiding the problem of the confidence intervals.
- ▷ The results provided are difficult to interpret and manage. Thus, how to exploit these results from the decision-maker's point of view remains an open issue, which needs to be addressed in future work.
- ▷ Very large memory demand and computational times are required. Table 4.1 reports the computational times in case of 2000 samples and 8000 combinations of values of the uncertain parameters. However, given that Matlab is an interpreted programming language, a tool developed in other environments may perform better. This issue will be tackled in future work.

Further research effort needs to be spent in order to verify whether the procedure illustrated here can be interpreted as a conservative counterpart to the calculus of probabilistic variables under stochastic independence, due to the dependence between the choice of confidence levels.

Parameters	Values
Number of FRVs	2000
Number of combinations of uncertain parameters	8000
CPU time (Intel Core 2 duo, 3.17 GHz, 2GB RAM)	≈ 30h

Table 4.1 – *Computational requirements of FRVs-based method*

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6 allée Émile Monso – BP 34038
31029 Toulouse cedex 4
France

Telephone: +33 534 32 32 00
Twitter: @TheFonCSI
Email: contact@FonCSI.org

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6 allée Émile Monso
ZAC du Palays - BP 34038
31029 Toulouse cedex 4

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